

Часть 17. Тригонометрические уравнения

Решите уравнения (17.1 — 17.27):

17.1. а) $2 \cos^2 x + 5 \sin x - 4 = 0$; б) $6 \cos^2 x + 5 \sin x - 7 = 0$; в) $\sin x - 2 \cos 2x = 1$; г) $3 \sin^2 2x + 7 \cos 2x - 3 = 0$.

17.2. а) $\sin x + \cos x = 0$; б) $\sin x = 2 \cos x$; в) $\sin^2 x - 2 \sin x \cos x = 3 \cos^2 x$; г) $6 \sin^2 x + \sin x \cos x - \cos^2 x = 2$; д) $\cos^2 x - 3 \sin x \cos x = \sin \frac{3\pi}{2}$.

17.3. а) $\sin x = \sin 5x$; б) $\cos 2x = \sin 5x$; в) $1 + \cos x + \cos 2x + \cos 3x = 0$; г) $\cos 5x + \cos 7x = \cos(\pi + 6x)$; д) $\sin 3x + \sin 5x = \sin 4x$; е) $1 - \cos(\pi + x) - \sin \frac{3\pi + x}{2} = 0$; ж) $\sin 3x = 2 \sin x$.

17.4. а) $\sin 2x \sin 6x = \cos x \cos 3x$; б) $\sin x \sin 3x + \sin 4x \sin 8x = 0$;

в) $\sin x \cos 2x + \cos x \cos 4x = \sin \left(\frac{\pi}{4} + 2x \right) \sin \left(\frac{\pi}{4} - 3x \right)$; г) $\cos 3x \cos 6x = \cos 4x \cos 7x$.

17.5. а) $\sin x - \cos x = 1$; б) $4 \sin x + 3 \cos x = 5$; в) $\sin x - 4 \cos x = 4$; г) $2 \sin x - \cos x = \frac{2}{5}$.

17.6. а) $\sin^2 2x + \sin^2 3x + \sin^2 4x + \sin^2 5x = 2$; б) $\cos^2 x + \cos^2 2x - \cos^2 3x - \cos^2 4x = 0$;

в) $\cos^2 \frac{x}{2} + \cos^2 \frac{3x}{2} - \sin^2 2x - \sin^2 4x = 0$; г) $\sin^2 3x + \sin^2 4x = \sin^2 5x + \sin^2 6x$.

17.7. а) $\sin 2x = \cos^4 \frac{x}{2} - \sin^4 \frac{x}{2}$; б) $\sin^4 x + \cos^4 x = \frac{5}{8}$; в) $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$;

г) $\sin^4 x + \cos^4 x - 2 \sin 2x + \frac{3}{2} \sin^2 2x = 0$; д) $\sin^4 x + \cos^4 x = \cos 4x$.

17.8. а) $\operatorname{tg} x + \operatorname{ctg} x = 2$; б) $1 + (1 + \sqrt{2})(\sin x + \cos x) + \sqrt{2} = \sin 2x$; в) $\sin^3 x - \cos^3 x = 1 + \sin x \cos x$;

г) $\sin 2x + 5(\sin x + \cos x) + 1 = 0$; д) $(1 - \sin 2x)(\cos x - \sin x) = 1 - 2 \sin^2 x$; е) $\operatorname{ctg} x - \operatorname{tg} x = \frac{\cos x - \sin x}{0,5 \sin 2x}$.

17.9. а) $\sin(15^\circ + x) + \sin(45^\circ - x) = 1$; б) $\cos(20^\circ + x) + \cos(100^\circ - x) = \frac{1}{2}$; в) $\operatorname{tg}(70^\circ + x) + \operatorname{tg}(20^\circ - x) = 2$;

г) $\operatorname{tg}(35^\circ + x) \operatorname{ctg}(10^\circ - x) = \frac{2}{3}$; д) $\operatorname{tg} x + \operatorname{tg} 50^\circ + \operatorname{tg} 70^\circ = \operatorname{tg} x \operatorname{tg} 50^\circ \operatorname{tg} 70^\circ$.

17.10. а) $\sin^{2012} x + \cos^{2012} x = 1$; б) $\sin^2 x + \sin^2 3x = 0$; в) $\sin x + \cos 7x = 2$; г) $\cos 4x \cos 5x = 1$.

17.11. $\sin 2x + 2 \operatorname{ctg} x = 3$. 17.12. $\cos 3x - \sin x = \sqrt{3}(\cos x - \sin 3x)$. 17.13. $2 \sin 4x + 16 \sin^3 x \cos x + 3 \cos 2x - 5 = 0$.

17.14. $2 \cos x(2 \sin x - 1) = -2 + 3 \sin x + 2 \sin^2 x$. 17.15. $1 + 2 \cos 3x \cos x - \cos 2x = 0$.

17.16. $4 \cos^3 x + 3\sqrt{2} \sin 2x = 8 \cos x$. 17.17. $8 \sin^4 x + 13 \cos 2x = 7$.

17.18. $2 \cos 2x + \cos^2 \frac{x}{2} - 10 \cos \left(\frac{5\pi}{2} - x \right) + \frac{7}{2} = \frac{1}{2} \cos x$.

17.19. $\sqrt{2} \cos \left(\frac{x}{5} - \frac{\pi}{12} \right) - \sqrt{6} \sin \left(\frac{x}{5} - \frac{\pi}{12} \right) = 2 \sin \left(\frac{x}{5} + \frac{2\pi}{3} \right) - 2 \sin \left(\frac{3x}{5} + \frac{\pi}{6} \right)$.

17.20. $\cos \left(x - \frac{\pi}{6} \right) + \sin \left(x - \frac{\pi}{3} \right) + \cos 2x = 1$. 17.21. $\operatorname{ctg}^4 x = \cos^3 2x + 1$. 17.22. $\sin x + \sin^2 x + \cos^3 x = 0$.

17.23. $2 \sin x + 8 \cos x \cos 2x = \sin 2x (\cos x - \sin x)$. 17.24. $\operatorname{tg} 2x + \sin 2x = \frac{8}{3} \operatorname{ctg} x$.

17.25. $\cos^4 x = \frac{1}{4} \cos 2x + \frac{1}{2} \cos^2 x \cos 8x$. 17.26. $\sin x + 2 \sin 2x = 3 + \sin 3x$. 17.27. $2 \operatorname{tg} 3x - 3 \operatorname{tg} 2x = \operatorname{tg}^2 2x \operatorname{tg} 3x$.

Решите системы уравнений (17.28 — 17.32):

17.28. $\begin{cases} 3 \sin 3x + \cos y = -4, \\ x + y = \frac{3\pi}{2}. \end{cases}$

17.29. $\begin{cases} x + \sin(x + y) = \frac{3}{2}, \\ 3x - \sin(x + y) = \frac{5}{2}. \end{cases}$

17.30. $\begin{cases} \sin x + \cos y = 1, \\ \cos 2x - \cos 2y = 1. \end{cases}$

17.31. $\begin{cases} 3 \cos x \cos y + 7 \sin x \sin y = 4, \\ 5 \cos x \cos y - 3 \sin x \sin y = 3. \end{cases}$ 17.32. $\begin{cases} 4 \sin y - 6\sqrt{2} \cos x = 5 + 4 \cos^2 y, \\ \cos 2x = 0. \end{cases}$

Решите неравенства (17.33 — 17.39):

17.33. $4 \sin^2 x \geq 1$. 17.34. $\sin x > \cos x$. 17.35. $2 \sin^2 x + 3 \cos x \geq 0$. 17.36. $\sin x - \cos x \leq \frac{3}{2}$. 17.37. $\operatorname{tg} x \geq \operatorname{ctg} x$.

17.38. $(\cos x \sin 2x - \sin x)(4 \sin x + 2 \cos x + 5) < 0$. 17.39. $\frac{\sin x + \sin 5x}{\sin x} \geq 0$.